

# Pressure independence of granular flow through an aperture

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We experimentally demonstrate that the flow rate of granular material through an aperture is controlled by the exit velocity imposed to the particles and not by the pressure at the base, contrary to what is often assumed in previous works. This result is achieved by studying the discharge process of a dense packing of monosized disks through an orifice. The flow is driven by a conveyor belt. This two-dimensional horizontal setup allows to uncouple pressure and velocity and, therefore, to independently control the velocity at which the disks escape the horizontal *silo* and the pressure in the vicinity of the aperture. The flow rate is found to be directly proportional to the belt velocity, independent of the amount of disks in the container and, thus, independent of the pressure in the outlet region. In addition, this specific experimental configuration makes it possible to get information on the system dynamics from a single image of the disks that rest on the conveyor belt after the discharge.

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The flow of granular media through an orifice presents some interesting features that have been intensely studied in the last 50 years [1–11]. What mainly differentiates the discharge of a container filled with granular matter from one filled with a viscous liquid is that the mass flow-rate does not depend on the height  $h$  of material above the outlet. The explanation most frequently used for this independence is based on the Janssen's effect: the distribution of the weight of the material onto the silo walls, due to the friction forces, leads to a saturation of the pressure at the bottom, which results in a constant flow-rate [12]. In such a reasoning, the pressure is thus implicitly assumed to govern the flow rate.

Here we show, by using an experimental setup in which the exit velocity of the grains is decoupled from the bottom pressure, that the above argument is improper. Indeed, different flow-rates can be achieved for the same bottom pressure; moreover, the flow-rate remains constant even if the pressure decreases during the discharge.

In general, the discharge of a silo through an orifice can present three regimes: a continuous flow, an intermittent flow, or a complete blockage of the flow due to arching [13–15]. In the continuous flow regime, the mass flow rate  $W$  is described by the so called Beverloo's law [1, 16]:  $W = C\rho_{3D}\sqrt{g}(A - kD)^{5/2}$  where  $A$  is the size of the opening,  $g$  the acceleration due to gravity,  $\rho_{3D}$  the bulk density and  $D$  the diameter of the granules whereas  $k$  and  $C$  are two empirical dimensionless constants. In a two-dimensional (2D) setup—or similarly in slit shaped apertures—one expects  $W = C\rho_{2D}\sqrt{g}(A - kD)^{3/2}$  [16]. In the jamming regime, the jamming probability is controlled by the ratio  $A/D$  of the aperture size to the grain diameter [14, 15, 17–20].

The discharge of particulate systems is not only found under the influence of gravity. In many industrial appli-

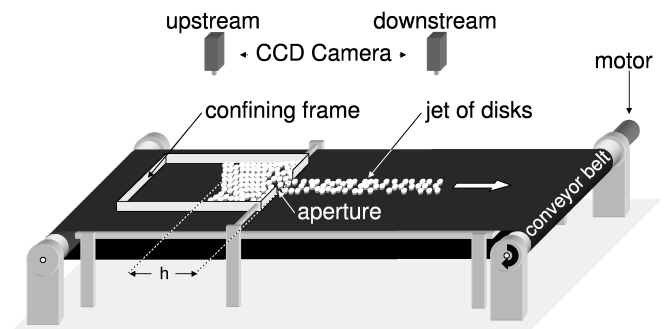


FIG. 1. Sketch of the experimental setup—Disks, placed on a conveyor belt which moves at a constant velocity  $V$  (white arrow), are forced to flow through an aperture in the confining frame. Observation of the grains—either remaining in the confining frame or forming the *jet* outside—makes it possible to study the dynamical properties of the discharge.

cations, the grains are horizontally transported at constant velocity (as on a conveyor belt [21] or floating on the surface of a flowing liquid [22]). Here, we analyze the discharge through an orifice of a 2D pile of disks driven at a constant velocity by a horizontal conveyor belt.

The experimental setup (Fig. 1) consists of a conveyor belt (width 40 cm, length 1 m) above which a confining Plexiglas frame (width 26 cm, length 54 cm) is maintained at a fixed position in the frame of the laboratory. A motor drives the belt at a constant velocity  $V$  which ranges from 0.4 cm/s to 4 cm/s. Downstream, the Plexiglas frame exhibits, at the center, a sharp aperture (width  $A \leq 10$  cm), much wider than the thickness of the walls (0.6 cm). The granular material consists of  $N = 450$  Plexiglas disks [thickness  $e = (3.1 \pm 0.1)$  mm, and diameter  $D = (1.04 \pm 0.01)$  cm]. Thus, in our experimental conditions, the ratio  $A/D \leq 10$ .

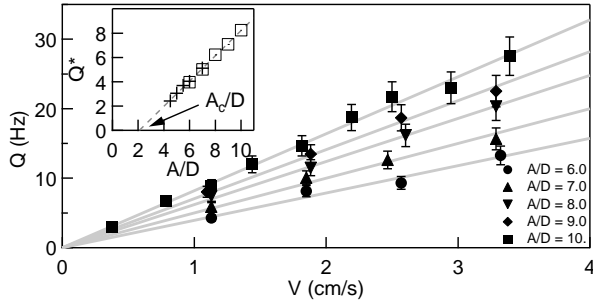


FIG. 2. Dependence of the flow rate  $Q$  on the belt velocity  $V$  for different apertures  $A/D$  (Average of 4 runs). Inset: The dimensionless slope  $Q^* \equiv QD/V$  obtained fitting data reported in the main panel (open squares) and/or from the probability distributions  $P(Q^*)$  (crosses) reported in Fig. 4 (errors are  $\pm 5\%$ ). The flow rate is expected to vanish for the critical flow rate,  $A_c$ .

Before the flow is started, the initial state of the system is obtained by depositing, in a disordered manner,  $N$  disks on the conveyor belt inside the confining frame. The belt is then moved at a low constant velocity until all grains are packed against the downstream wall, whose aperture is still kept closed ( $A = 0$ ). The packing fraction of this initial configuration is  $\Phi \approx 0.82$ . Then, the aperture is opened to the desired width  $A \neq 0$  and the belt is moved at constant velocity  $V$ . The belt is stopped whenever the Plexiglas frame is emptied or the flow ceases due to the formation of a jamming arch.

A video camera (Pixelink, PL-A741) is used to image the system from above. We record the evolution of the discharge process in either of the following ways: Images of disks that remain inside the confining frame (upstream) are acquired every second during the discharge; One single image of the *jet* of disks that are at rest on the belt outside the confining box (downstream) is acquired after the discharge.

An intensity threshold is used to convert the pictures into binary images. The number  $N_i$  of disks that remain inside the confining frame at time  $t$  can be calculated from the number of white pixels in the images acquired upstream [Area of a disk ( $733 \pm 8$ ) pixels]. The instantaneous disk flow-rate (averaged over 1 s, because of the acquisition frequency) is defined as  $Q \equiv -\frac{dN_i}{dt}$ . When the aperture is large ( $A/D \geq 6$ ), the flow-rate is rather continuous throughout the discharge,  $N_i$  depends linearly on time and, for any given  $A$ , the flow rate is proportional to the belt velocity (Fig. 2). This indicates that, in our experimental conditions, the granular bed rearranges in a time which is smaller or compares with the time necessary for the grains to escape the outlet region. Indeed, if a significant relaxation time is associated with the rearrangement of the granular bed, above a critical velocity, a change in the drive is barely followed by the system and a qualitative change in  $Q(V)$  is observed [21]. In the inset in Fig. 2, we report the dimensionless slope  $Q^* \equiv QD/V$  as a function of the dimensionless aperture  $A/D$  and get,

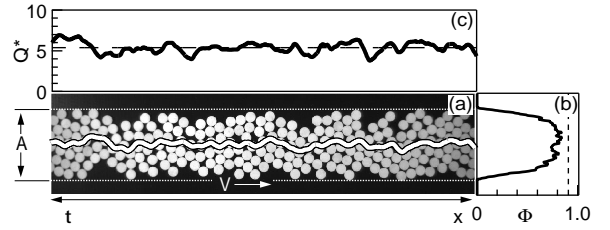


FIG. 3. Image of the *jet* and the corresponding density profiles. (a) Image of the jet after the discharge of the container for  $A/D = 7.0$ . (b) Area fraction profile  $\Phi$  in the perpendicular direction. Each data point is obtained as the fraction of white pixels in the corresponding row after thresholding [the vertical dashed line corresponds to the maximum attainable density  $\pi/(2\sqrt{3})$ ]. (c) Instantaneous dimensionless flow rate  $Q^*$  against the longitudinal coordinate  $x$ . The flow rate is obtained from the fraction of white pixels in the corresponding column  $1D$  wide.

for  $A/D \geq 6$ , the empirical law:

$$Q = C \frac{V}{D} (A/D - k) \quad (1)$$

where  $k = 2.1 \pm 0.2$  and  $C = 1.04 \pm 0.02$ . The flow rate is thus expected to vanish at a value  $A_c \equiv kD$  of the aperture. The value of the constants  $k$  and  $C$  will be discussed below (see comment on Ref. [21]). Note that the flow rate is not well defined and cannot be extracted directly from  $N_i(t)$  for  $A/D < 6$ , the system being then likely to jam.

In order to estimate the flow rate for  $A/D < 6$ , it is particularly interesting to consider the second measurement method in which we take a single picture of the jet at rest on the belt after the experiment is stopped (Fig. 3a). After being released from the confining frame, the grains are simply advected with velocity  $V$  and the belt is long enough so that all disks remain on it after the discharge. Thus, the distribution of disks on the belt provides information on the outflow of grains ever since its onset. Therefore, such an image provides the full history of the flow rate: the spatial coordinate along the belt longitudinal axis can be translated into time. Moreover, since the flow rate  $Q$  is proportional to  $V$  (Fig. 2), one can limit the study to a single value of the velocity.

Let us first comment on the intensity profile in the direction perpendicular to the jet (Fig. 3b): the fraction of white pixels in a row (one pixel wide and about 1500 pixels long) corresponds to the average density  $\Phi$  of the disks in this row. The profile is smooth and, even for an aperture as large as seven disk diameters, there is no well defined plateau. Such boundary effects, which are present in a large part of the profile, are responsible for the effective aperture width,  $(A - A_c)$ , pointed out by the finite value of  $k$  in Eq. (1).

We now consider the intensity profile along the longitudinal axis of the jet. In order to define an instantaneous flow rate, we arbitrarily integrate over a 30-pixel-wide window (a disk diameter) around the considered

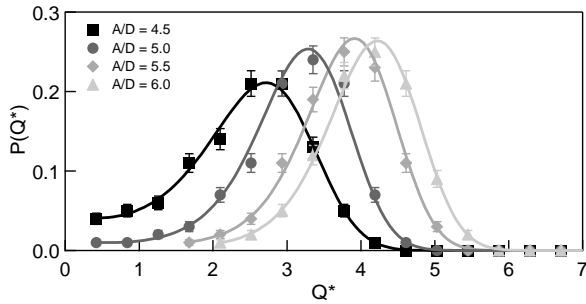


FIG. 4. Probability distribution of the dimensionless flow rate  $Q^*$  for different values of  $A/D$ . For each aperture  $A/D$ , the probability is calculated from all the values of the flow-rate measured during a set of 5 to 10 experiments. For the largest aperture reported ( $A/D = 6.0$ ), the probability distribution is close to Gaussian, whereas a relatively large probability of small  $Q^*$  values appears when  $A$  is decreased. The continuous lines are only a guide to the eye.

position. The dimensionless flow rate  $Q^*$  is obtained by counting the number of white pixels in a column of width  $D$  along the jet and dividing it by the surface area of one disk. Other window widths were tested: fluctuations are smaller for wider windows but the average value of  $Q^*$  remains the same to within 0.2%. The example reported in Fig. 3c shows that, even for a large aperture,  $Q^*$  is subjected to significant fluctuations (standard deviation around 15% and difference between extreme values of about 60% of the average). The probability distribution  $P(Q^*)$  is shown in Fig. 4. On the one hand,  $P$  is almost Gaussian when the aperture is large ( $A/D > 6$ ). The probability of small  $Q^*$  is negligible and the flow is continuous. On the other hand, when the size of the aperture is reduced, the system might jam ( $Q^* = 0$ ) transiently or permanently (in agreement with similar experiments carried out in a 2D vertical silo [15, 23]). In accordance, the probability of small  $Q^*$  increases.

From the probability distribution, it is possible to define a mean flow rate even when the system presents intermittent jams. The average flow rate during the effective flow events can be estimated by defining  $\bar{Q}^* \equiv \int Q^* P(Q^*) dQ^*$  (Note that jamming events correspond to a peak at  $Q^* = 0$  that we disregard in our analysis in order to account for the flow events only). The estimations of  $\bar{Q}^*$  obtained in this way for large values of  $A/D$  are in full agreement with measurements obtained from the upstream technique (Fig. 2, inset). In addition, the data obtained for smaller apertures also obey to Eq. (1), which indicates that there is no drastic change in the empirical law in presence of intermittency. However, we mention that, in 3D configurations, a deviation from the 5/2 Beverloo's scaling for very small apertures has been observed [13]. Thus, a deviation from the linear relation (1) may be present for  $A/D < 4.5$ .

We emphasize that, in our experimental conditions, the effective pressure exerted by the granular bed on the downstream wall depends on the dynamical friction force

between the disks and the belt. So, at a constant velocity  $V$ , the bottom pressure continuously decreases as the number of disks inside the system decreases during the discharge. Thus, during the whole discharge, a constant flow-rate  $Q$  is achieved while the pressure continuously varies, which indicates that the granular flow rate is not controlled by it. The horizontal configuration makes it possible to impose the exit velocity and, moreover, to tune independently the overall pressure by changing the weight of the disks or their initial number inside the frame. In order to measure the flow-rate in a different pressure range for the same output velocity, we added an individual extra weight on top of each disk. We thus increased the pressure by increasing the friction force acting on the disks without changing any other property of the system. We did not observe any change in the flow-rate in spite of the change in the pressure, which is again in agreement with a flow-rate only controlled by the escape velocity.

At this point, it is particularly interesting to discuss the physical ingredients leading to Beverloo's law. In our experimental conditions, we obtain a simple linear law relating the flow-rate to the aperture width  $A$ , whereas the corresponding relation in a 2D gravity driven configuration corresponds to  $Q \propto A^{3/2}$ . Both relations can be derived from a dimensional analysis, taking into account the estimation of the velocity  $v$  at which the grains escape the system. In the case of the 2D vertical silo, the *free falling arch* assumption [24] leads to  $v \propto \sqrt{gA}$ . Hence, considering the mass flow rate through an aperture of size  $A$ ,  $W \equiv \rho_{2D} A v$  leads to  $W \propto \rho_{2D} \sqrt{g} A^{3/2}$ . Boundary effects at the aperture edges lead to two boundary layers of thickness  $\sim D$  (the so-called *empty annulus* [25]). Considering the effective aperture size  $A - kD$  instead of  $A$ , one gets  $Q \propto (A/D - k)^{3/2}$ . The same arguments lead to the Beverloo's law in 3D. In the case of the conveyor belt  $v = V$  and the account of the empty annulus leads to  $Q \propto V(A/D - k)$  in agreement with Eq. (1). It is then worth mentioning that, in the limit of infinitely large apertures, the correction due to the boundary layers can be neglected and  $Q \rightarrow \rho A V$  ( $\rho$  is the number of disks per unit area) so that  $C = \rho D^2$  and, hence,  $Q = \rho V D (A/D - k)$ . Taking into account  $\Phi = \rho \frac{\pi D^2}{4}$  ( $\Phi$  is the area fraction), we obtain that  $C = \frac{4}{\pi} \Phi$ . From the experimental value  $C = 1.13 \pm 0.02$ , we estimate  $\Phi = 0.82 \pm 0.02$ , which is indeed compatible with the area fraction initially measured inside the confining frame and which does not noticeably vary during the discharge. We remark that the dimensional analysis could be based on the pressure since it also exhibits a time-scale. In the case of a 3D silo, denoting  $P$  the pressure at the outlet, one gets  $W \propto \sqrt{\rho_{3D} P} A^2$ . Be the pressure limited by Janssen's effect or not,  $W$  scales with  $A^2$ , which contradicts the experimental  $A^{5/2}$  scaling.

In most previous works [12], the independence of the flow rate on the silo height  $h$  is explained by means of two postulates. The first one is: if the height is larger than twice the diameter of the silo, the pressure at the

bottom is constant (Janssen's effect). The second one is: if during the discharge the pressure at the bottom is constant, the flow rate is, as a consequence, also constant. We have proven that this second postulate is not valid. Indeed, experimentally, for the same pressure in the outlet region (same amount of disks inside the system), the flow rate is proportional to the belt velocity. Moreover, the same flow-rate is obtained with different values of the pressure in the outlet region (different amount of disks inside the system or additional weights on the disks). In addition, theoretically, the physical ingredients used to establish Beverloo's law, which is correctly satisfied experimentally, do not involve the local pressure but a characteristic velocity. Thus, Beverloo's law already predicts that the flow-rate should be independent of  $h$  even when the Janssen's effect is not at stake.

In conclusion, using an experimental setup which enables to control the particle velocity independently from the other parameters of the flow, we have shown that the granular flow rate through an orifice is not controlled by the local pressure and that invoking the Janssen's effect is not pertinent to explain the constant flow-rate mea-

sured during a silo discharge. The flow rate is controlled by the mechanism driving the grains out of the system and by the geometry of the outlet, features which govern the exit velocity, and not by the pressure upstream. For applications, our findings thus suggest that granular flow rates can be increased, in gravity driven systems, by locally increasing the particle velocities at the outlet, which might be achieved with devices having a low energetic cost. From the fundamental point of view, they show that more effort must certainly be done to verify some misleading assertions widely stated in material science of dissipative particles during the last five decades.

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